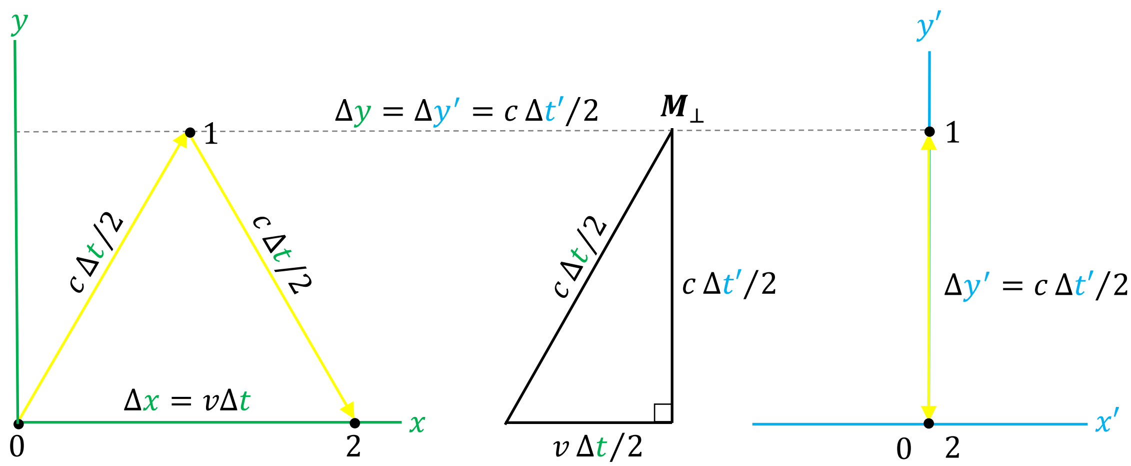
Special Relativity Time Dilation

Derivation of the time dilation formula using the light path reflected off mirror $\boldsymbol{M}_\perp$ which is offset from the light source perpendicular to the direction of motion (example shown has $v = c/2$)

event 0: *send light pulse*
event 1: *bounce at mirror* $\boldsymbol{M}_\perp$
event 2: *receive light pulse (also send next light pulse)*



S' moves at constant speed v in the $+x$ direction relative to S
S and S' perpendicular distances Δy and $\Delta y'$ are equal (no length contraction)
let $\Delta t = S$ time from event 0 to event $2 =$ one S clock tick
let $\Delta t' = S'$ time from event 0 to event $2 =$ one S' clock tick
let $\Delta y'$ be S' (proper) distance from event 0 to event $1 = c\,\Delta t'/2$
S distance from event 0 to midway between events 0 and $2 = v\,\Delta t/2$
S light path (diagonal) distance from event 0 to event $1 = c\,\Delta t/2$

use Pythagorean formula: $(c\,\Delta t/2)^2 = (v\,\Delta t/2)^2 + (c\,\Delta t'/2)^2$

$$c^2(\Delta t)^2 = v^2(\Delta t)^2 + c^2(\Delta t')^2$$
$$(c^2 - v^2)(\Delta t)^2 = c^2(\Delta t')^2$$
$$(\Delta t)^2 = \frac{c^2}{c^2 - v^2}(\Delta t')^2 = \left(1 - \frac{v^2}{c^2}\right)^{-1}(\Delta t')^2$$
$$\Delta t = \left(1 - \frac{v^2}{c^2}\right)^{-1/2}\Delta t'$$

$$\boxed{\Delta t = \gamma \Delta t'}$$

$\gamma \geq 1 \Rightarrow$ time dilation as seen by S frame:
S time duration Δt is greater (by a factor of γ) than S' time duration $\Delta t'$
S sees S' clocks tick slower than S clocks (by a factor of γ)